Brane Gases and Stabilization of Shape Moduli with Momentum and Winding Stress

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Abstract

In a toy model with gases of membranes and strings wrapping over a two-dimensional internal torus, we study the stabilization problem for the shape modulus. It is observed that winding modes of partially wrapped strings and momentum modes give rise to stress in the energy momentum tensor. We show that this stress dynamically stabilizes the shape modulus of the two torus.

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I. INTRODUCTION

In usual Kaluza-Klein picture, the extra dimensions predicted by string/M theory are assumed to be compact and small. The four dimensional low energy content of the theory (like the spectrum of light particles, the strengths of the coupling constants etc.) is determined by the internal space. Although phenomenologically viable models can be obtained in Calabi-Yau or G2 manifold compactifications, in its current formulation of string/M theory there is no principle which would single out a specific compact space. This is a major problem which prevents one to test the validity of string/M theory by low energy phenomena.

Besides the vacuum selection problem, one should also confront the issue of the cosmological evolution of the extra dimensions. It is well known that there is a limit to the change in the size of the internal space following the epoch of big bang nucleosynthesis [1]. Moreover, the size and the shape of the compact space are parametrized by moduli scalar fields which are in general dynamical. Therefore, to avoid any conflict with observations, moduli fields should somehow be fixed in string/M theory.

Brane gas cosmology, which was initiated in [2] and then developed in [3], may offer a solution to the stabilization problem in a cosmological context.[1] In [26–28], it is found that string winding and momentum modes can stabilize the radii of an internal torus[2]. Moreover, as shown in [29, 30], this mechanism survives in the presence of linearized cosmological perturbations. In [32, 33] we generalize these results to higher dimensional branes and find that the volume modulus of an internal Ricci flat manifold can be stabilized by a gas of branes wrapping over it. This extension is important for two reasons. Firstly winding strings do not exist in the spectrum when the first homology class of the internal space is zero and secondly they may not be able stabilize volume modulus when the topology is not equal to the product of 1-cycles. In [34], we also show that long-wavelength cosmological perturbations does not alter this result.

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[1] Different aspects of brane gas cosmology are studied in the literature: T-duality invariance, a possible loitering phase, generalization to curved Ricci flat manifolds and M-theory are discussed in the papers [4], [5], [6] and [7], respectively. Cosmological solutions with brane gas sources are constructed in [8–14]. Annihilation of winding branes which is crucial for the Brandenberger and Vafa decompactification mechanism is studied in [15–20]. Inflation in the context of brane gas cosmology is considered in [21, 22] and the stabilization of extra dimensions in a democratic wrapping scheme is demonstrated in [23–25].

[2] See [35] for some concerns about dilaton stabilization by string gases and [36] for an alternative approach to stabilization.

In a recent paper [37], the stabilization of extra dimensions is reconsidered in the presence of a string gas carrying two-form flux and it is found that, in addition to volume modulus, the flux and the shape moduli are also dynamically stabilized. In this paper, we address the stabilization problem for the shape modulus without introducing any flux. Following our earlier work [32], we consider a toy model in Einstein gravity with membrane and string gases wrapping over an internal two-torus, but this time we take shape to be dynamical. In that case, we observe that winding modes of partially wrapped branes (strings in this model) and momentum modes give rise to stress in the energy momentum tensor. We will show that this stress is capable of dynamically stabilizing the shape modulus.

As for the size moduli, we will see that when the observed space is three dimensional the winding and the momentum modes stabilize them as in [32]. When the dimension is greater than three, there appears a complication which was pointed out earlier in [9], i.e. partially wrapped branes force the transverse compact directions to expand. For stabilization of the size moduli in that case, one should restrict the relevant physical parameters so that this does not produce an instability. (In [38], which appeared while our work was being finalized, it is also found that special massless string modes can stabilize volume, shape and dilaton moduli.)

The organization of the paper is as follows. In the next section we calculate the energy momentum tensors and obtain a background solution corresponding to a fixed rectangular torus and an expanding observed space. In section III, we study the linearized fluctuations around the background geometry and find that the shape modulus is dynamically stabilized. In this section, we also discuss the stabilization of the size moduli. In section IV, we conclude with brief remarks and future directions.

II.   ENERGY MOMENTUM TENSORS AND THE BACKGROUND SOLUTION

We consider an $(m + 3)$-dimensional space-time with the metric

$$ds^2 = -dt^2 + e^{2B} \, dx^i dx^i + G_{ab} \, dy^a dy^b, \tag{1}$$

where $i, j = 1, .., m$ and $a, b = 1, 2$. The m-dimensional observed space is spanned by x^i and $y^a = (y^1, y^2)$ are coordinates on an internal two-torus T^2 so that $y^a \sim y^a + 2\pi$. In a

cosmological setup one should take

$$B = B(t), \qquad G_{ab} = G_{ab}(t). \tag{2}$$

A convenient parametrization of the metric on T^2 is

$$G_{ab} = \begin{bmatrix} R_1^2 & R_1 R_2 \sin\theta \\ R_1 R_2 \sin\theta & R_2^2 \end{bmatrix}, \tag{3}$$

where R_1 and R_2 are the radii of y^1 and y^2, respectively, and θ is the shape modulus. Note that $\theta = 0$ corresponds to a rectangular torus. We choose

$$e^{\hat{a}}{}_b = \begin{bmatrix} R_1 \cos(\theta/2) & R_2 \sin(\theta/2) \\ R_1 \sin(\theta/2) & R_2 \cos(\theta/2) \end{bmatrix}, \tag{4}$$

as an orthonormal basis in the tangent space (the hatted and unhatted indices refer to the tangent and coordinate spaces, respectively).

Up to a certain constant prefactor, the energy of a membrane wrapping over T^2 is equal to

$$E_w = \sqrt{\det G}. \tag{5}$$

In the observed space the winding energy is concentrated at the position of the membrane and this gives a delta function singularity. But for a gas of such branes the delta function is smoothed out in the continuum fluid approximation [9]. The energy spectrum of the momentum modes corresponding to small vibrations of a membrane on T^2 is equal to the eigenvalues of the Laplacian of the metric G_{ab} which are characterized by two integers $n_a = (n_1, n_2)$. The energy of a mode described by the wave-function $e^{iy^a n_a}$ is given by

$$E_m = \sqrt{G^{ab} n_a n_b}. \tag{6}$$

Note that E_m is equal for two modes having quantum numbers n_a and $-n_a$.

For strings wrapping over y_1 and y_2, the winding energy is equal to

$$E_s = \sqrt{G_{11}} + \sqrt{G_{22}}. \tag{7}$$

As in the membrane case, the energy is distributed smoothly for a gas in the continuum approximation. String momentum modes,[3] which are labeled by $n_a = (n_1, n_2)$ with $n_1 = 0$

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[3] Here we consider semiclassical quantization of strings in a (non-covariant) physical gauge. Thus the tachyon is absent in the spectrum and there appears no critical dimension.

or $n_2 = 0$, can be viewed as a subset of membrane momentum modes and their energy is still given by (6). The corresponding wave-functions are $e^{iy^1 n_1}$ and $e^{iy^2 n_2}$, which represent small vibrations of strings wrapping over y^1 and y^2, respectively.

From (5), (6) and (7) the energy densities can be calculated as

$$\rho = \frac{E}{\text{Vol}_S}, \tag{8}$$

where $\text{Vol}_S = e^{mB}\sqrt{\det G}$ is the total spatial volume. It is well known that (see e.g. [39]) one can write a matter Lagrangian

$$\mathcal{L}_m = -2\rho \tag{9}$$

for an energy density ρ. Coupling $\mathcal{L}_m$ to Einstein-Hilbert action

$$S = \int \sqrt{-g} \left[R + \mathcal{L}_m \right], \tag{10}$$

one obtains the field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}, \tag{11}$$

where

$$T_{\mu\nu} = -\frac{1}{\sqrt{-g}} \frac{\partial}{\partial g^{\mu\nu}} \left[\sqrt{-g} \mathcal{L}_m \right] \tag{12}$$

is the energy momentum tensor.

Using (5) in the above formulas, the energy momentum tensor for membrane winding modes can be found as

$$
\begin{aligned}
T_{00} &= N_w e^{-mB}, \\
T_{ij} &= 0, \\
T_{ab} &= N_w e^{-mB} G_{ab},
\end{aligned}
\tag{13}
$$

where N_w is a numerical factor containing the tension and the number density of membranes. For string winding modes (7) implies

$$
\begin{aligned}
T_{00} &= e^{-mB} \left[N_{s1} \frac{G_{11}}{\sqrt{\det G}} + N_{s2} \frac{G_{22}}{\sqrt{\det G}} \right] \\
T_{ij} &= 0, \\
T_{ab} &= -e^{-mB} \left[N_{s1} \frac{G_{1a} G_{1b}}{\sqrt{G_{11}}\sqrt{\det G}} + N_{s2} \frac{G_{2a} G_{2b}}{\sqrt{G_{22}}\sqrt{\det G}} \right],
\end{aligned}
\tag{14}
$$

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where N_{s1} and N_{s2} are proportional to the tension and the number densities of strings wrapping over y^1 and y^2, respectively. Finally, the energy momentum tensor for a gas of string and membrane momentum modes with fixed n_a can be obtained from (6) as

$$
\begin{aligned}
T_{00} &= N_m \, e^{-mB} \frac{\sqrt{G^{ab} n_a n_b}}{\sqrt{\det G}}, \\
T_{ij} &= 0, \\
T_{ab} &= N_m \, e^{-mB} \frac{n_a n_b}{\sqrt{\det G} \sqrt{G^{cd} n_c n_d}},
\end{aligned}
\tag{15}
$$

where N_m is the number density. Note that (15) obeys $g^{\mu\nu} T_{\mu\nu} = 0$ and moreover $T_{ij} = 0$, therefore momentum modes behave like a gas of massless particles confined in the compact space. One can verify that all tensors (13), (14) and (15) are conserved

$$
\nabla_\mu T^{\mu\nu} = 0,
\tag{16}
$$

so that the field equations (11) are consistent.

It is clear from (5) and (7) that winding modes become heavy as T^2 expands and in thermal equilibrium they are expected to decay into lighter excitations. However, in usual brane gas scenario winding modes are assumed to fall out of thermal equilibrium, so one can take N_w in (13) and N_{s1}, N_{s2} in (14) as constant numbers. Similarly, from (6) the momentum modes become heavy as T^2 shrinks. Thus, for momentum modes to stop the contraction of the internal space, there should be a mechanism for them to survive the annihilation process. For a string gas (in dilaton gravity) the annihilation rates of the winding and the momentum modes are equal to each other by T-duality invariance of the field and the Boltzmann equations [19]. Therefore T-duality dictates that momentum modes should also fall out of thermal equilibrium like winding modes. Although the toy model we consider does not have T-duality invariance, the final scenario (obtained by including all possible excitations) is expected to have this symmetry [4]. Viewing the model here as a part of the complete picture, we assume that momentum modes also fall out of thermal equilibrium and thus N_m in (15) is a constant number density.

It is possible to give an alternative argument without invoking T-duality invariance that momentum modes should exist in the spectrum when the internal space contracts too much. Branes are dynamical objects and one would expect them to fluctuate when they are forced to collapse totally. This is crucial for stability since otherwise any unwrapped brane propagating in the bulk would undergo a complete collapse under the influence of its own tension.

For momentum and (partially wrapped) string winding modes, (14) and (15) imply non-vanishing stress[4] in the energy momentum tensor since in general $T_{\hat{a}\hat{b}} \neq 0$ when $\hat{a} \neq \hat{b}$. It is important to refer to an orthonormal basis in determining the existence of stress. For instance, the off diagonal components of the winding energy momentum tensor of membranes (13) is non-vanishing in the coordinate basis. As we will see, however, (totally wrapped) membrane winding modes do not play a role in the dynamics of the shape modulus.

To obtain the background solution we assume

$$B = B(t), \qquad G_{ab} = \text{const.} \tag{17}$$

Eq. (11) then yields

$$m\ddot{B} + m\dot{B}^2 = -\frac{(m-2)N_w}{(m+1)} e^{-mB} - \sum_i N_m^{(i)} \frac{\sqrt{G^{ab}n_a^{(i)}n_b^{(i)}}}{\sqrt{\det G}} e^{-mB}$$
$$-\frac{(m-1)N_{s1}}{(m+1)} \frac{\sqrt{G_{11}}}{\sqrt{\det G}} e^{-mB} - \frac{(m-1)N_{s2}}{(m+1)} \frac{\sqrt{G_{22}}}{\sqrt{\det G}} e^{-mB}, \tag{18}$$

$$\ddot{B} + m\dot{B}^2 = \frac{3N_w}{m+1} e^{-mB} + \frac{2N_{s1}}{(m+1)} \frac{\sqrt{G_{11}}}{\sqrt{\det G}} e^{-mB} + \frac{2N_{s2}}{(m+1)} \frac{\sqrt{G_{22}}}{\sqrt{\det G}} e^{-mB}, \tag{19}$$

$$\frac{(m-2)N_w}{(m+1)} G_{ab} + N_{s1} \frac{G_{1a}G_{1b}}{\sqrt{G_{11}}\sqrt{\det G}} + N_{s2} \frac{G_{2a}G_{2b}}{\sqrt{G_{22}}\sqrt{\det G}}$$
$$= \left[\frac{2N_{s1}}{(m+1)} \frac{\sqrt{G_{11}}}{\sqrt{\det G}} + \frac{2N_{s2}}{(m+1)} \frac{\sqrt{G_{22}}}{\sqrt{\det G}} \right] G_{ab} + \sum_i N_m^{(i)} \frac{n_a^{(i)}n_b^{(i)}}{\sqrt{G^{cd}n_c^{(i)}n_d^{(i)}}\sqrt{\det G}}, \tag{20}$$

where $N_m^{(i)}$ is the number density for the i'th momentum mode with quantum numbers $n_a^{(i)}$.

For a rectangular torus with $\theta = 0$ (i.e. $G_{12} = 0$) the energies of the modes with (n_1, n_2) and $(n_1, -n_2)$ are equal to each other. Therefore, these modes should have the same number density. This implies that the off-diagonal component of (20) is identically satisfied after the sum over the pairs (n_1, n_2) and $(n_1, -n_2)$. Taking moreover $R_1 = R_2 \equiv R_0$, the number densities of strings should also be the same, i.e. $N_{s1} = N_{s2} \equiv N_s$. Eq. (20) then gives

$$\frac{2(m-2)N_w}{(m+1)} R_0^3 + \frac{2(m-3)N_s}{(m+1)} R_0^2 = \sum_i N_m^{(i)} \sqrt{\delta^{ab}n_a^{(i)}n_b^{(i)}}. \tag{21}$$

Note that the cubic equation for R_0 has one unique positive root for $m > 2$, and for $m \leq 2$ the ansatz is not valid. From now on, we restrict the dimension of the observed space $m > 2$.

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[4] This stress was not observed in [32] since only the volume modulus was taken to be dynamical there.

Eq. (19) can be solved to get[5]

$$B(t) = \frac{2}{m} \ln(bt), \tag{22}$$

where

$$b^2 = \frac{3m\, N_w}{2(m+1)} + \frac{2m\, N_s}{(m+1)R_0}. \tag{23}$$

One can now check that (18) is identically satisfied. Therefore the metric

$$ds^2 = -dt^2 + (bt)^{4/m} dx^i dx^i + R_0^2\, dy^a dy^a \tag{24}$$

solves all field equations and describes a background supported by brane winding and momentum modes. The moduli fields of this solution are R_1, R_2 and θ, corresponding to the radii of y^1, y^2 and the angle between them. They have the vacuum values $R_1 = R_2 = R_0$ and $\theta = 0$. From the energy-momentum tensors (13)-(15), we see that the winding and the momentum modes apply negative and positive pressures along the compact directions, and they behave like pressureless dust in the observed space. Therefore, it is not surprising that in (24) the observed space expands exactly with the same power for pressureless dust and the internal directions are stabilized under the action of positive and negative pressures.

III. STABILIZATION OF THE MODULI

Having obtained the background solution, we now study linearized perturbations around it. The fluctuations are denoted by δB, δR_1, δR_2 and $\delta\theta$, which are taken to be the functions of time. We carry the calculation in the orthonormal basis (4). Variation of (13), which is the winding energy momentum tensor of membranes, gives

$$\delta T_{\hat{0}\hat{0}} = -m\, N_w\, e^{-mB}\, \delta B, \quad \delta T_{\hat{a}\hat{b}} = m\, N_w\, e^{-mB}\, \delta B\, \delta_{ab}. \tag{25}$$

From (14), for string winding modes we find

$$\delta T_{\hat{0}\hat{0}} = -\frac{2m\, N_s}{R_0} e^{-mB}\, \delta B - \frac{N_s}{R_0^2} e^{-mB}\, [\delta R_1 + \delta R_2]\,,$$

$$\delta T_{\hat{a}\hat{b}} = \frac{m\, N_s}{R_0} e^{-mB}\, \delta B\, \delta_{ab} + \frac{N_s}{R_0^2} e^{-mB}\, [\delta R_1\, \delta_{a2}\, \delta_{b2} + \delta R_2\, \delta_{a1}\, \delta_{b1}]$$

$$- \frac{N_s}{R_0} e^{-mB}\, [\delta_{a1}\, \delta_{b2} + \delta_{a2}\, \delta_{b1}]\, \delta\theta. \tag{26}$$

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[5] By defining a new time coordinate it is possible to obtain the most general solution of (19), which asymptotically becomes (22). Therefore, (22) describes the late time behavior we are interested in.

On the other hand, the perturbation of (15) for momentum modes yields

$$\delta T_{\hat{0}\hat{0}} = -m \frac{|\vec{n}| N_m}{R_0^3} e^{-mB} \delta B - \frac{N_m}{R_0^4 |\vec{n}|} e^{-mB} n_a n^b \delta e^a{}_b - \frac{|\vec{n}| N_m}{R_0^4} e^{-mB} \delta e^a{}_a \qquad (27)$$

$$\delta T_{\hat{a}\hat{b}} = -\frac{N_m}{R_0^4 |\vec{n}|} n_a n_b e^{-mB} \left[m R_0 \delta B + \delta e^c{}_c \right] + \frac{N_m}{R_0^4 |\vec{n}|^3} n_a n_b e^{-mB} n_c n^d \delta e^c{}_d$$
$$-\frac{2N_m}{R_0^4 |\vec{n}|} n_c n_{(b} \delta e^c{}_{a)}, \qquad (28)$$

where

$$\delta e^a{}_b = \begin{bmatrix} \delta R_1 & R_0 \delta\theta/2 \\ R_0 \delta\theta/2 & \delta R_2 \end{bmatrix}, \qquad (29)$$

$|\vec{n}| = \sqrt{n_1^2 + n_2^2}$ and all operations on indices are performed with the Kronecker delta function. Note that (27) and (28) are invariant under $n_a \to -n_a$.

The number densities of momentum modes are expected to be distributed according to Boltzmann weight. This does not contradict with the assumption that they fall out of thermal equilibrium, since just before decoupling they are expected to obey Boltzmann distribution and then evolve accordingly. Therefore it is sufficient to consider the modes having minimum energy, since the contributions of others are exponentially suppressed. In our case the minimum energy excitations have quantum numbers $n_a = (1,0)$, $n_a = (-1,0)$, $n_a = (0,1)$ and $n_a = (0,-1)$, which should have the same density N_m since they have the same energy. Note that both strings and membranes give rise to such momentum modes and N_m denotes the total number density.

Neglecting all other momentum modes, the common radius of the internal torus obeys

$$\frac{(m-2)N_w}{(m+1)} R_0^3 + \frac{(m-3)N_s}{(m+1)} R_0^2 = 2N_m. \qquad (30)$$

Rewriting the field equations in the form

$$R_{\hat{\mu}\hat{\nu}} = T_{\hat{\mu}\hat{\nu}} - \eta_{\hat{\mu}\hat{\nu}} \frac{T^{\hat{\lambda}}{}_{\hat{\lambda}}}{(m+1)}, \qquad (31)$$

and using (25)-(28), we find the following second order equations

$$\ddot{\delta B} + \frac{4}{t} \dot{\delta B} + \frac{2}{mt} \dot{\delta Y} = -\frac{m(3 N_w + 4 N_s)}{(m+1) R_0 (bt)^2} \delta B - \frac{2N_s}{(m+1) R_0 (bt)^2} \delta Y, \qquad (32)$$

$$\ddot{\delta Y} + \frac{2}{t} \dot{\delta Y} = \left[\frac{(m-3)N_s}{(m+1) R_0} - \frac{6N_m}{R_0^3} \right] \frac{1}{(bt)^2} \delta Y, \qquad (33)$$

$$\ddot{\delta Z} + \frac{2}{t} \dot{\delta Z} = -\left[\frac{N_s}{R_0} + \frac{2N_m}{R_0^3} \right] \frac{1}{(bt)^2} \delta Z, \qquad (34)$$

$$\ddot{\delta \theta} + \frac{2}{t} \dot{\delta \theta} = -\left[\frac{2N_s}{R_0} + \frac{4N_m}{R_0^3} \right] \frac{1}{(bt)^2} \delta\theta, \qquad (35)$$

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and the first order initial constraint

$$(m - 1)\delta\dot{B} + \delta\dot{Y} + \frac{(m-1)}{t}\delta B + \frac{(m-2)}{mt}\delta Y = 0, \tag{36}$$

where

$$\delta Y = \frac{1}{R_0}\left[\delta R_1 + \delta R_2\right], \tag{37}$$

$$\delta Z = \frac{1}{R_0}\left[\delta R_1 - \delta R_2\right]. \tag{38}$$

As a consistency check of these equations we verify that the time derivative of (36) is identically satisfied upon using the field equations (32) and (33). Note that δY can be solved from (33) which in turn can be used in (32) to fix δB. The perturbations δY and δZ are related to volume $R_1 R_2$ and fraction R_1/R_2 moduli.

Let us first consider the equation (35) for $\delta\theta$. Writing

$$\delta\theta = t^k \tag{39}$$

one finds that k obeys

$$k^2 + k + c = 0, \tag{40}$$

where $c = 2N_s/(b^2 R_0) + 4N_m/(b^2 R_0^3)$. The roots are

$$k = -\frac{1}{2} \pm i\,\Delta, \tag{41}$$

where

$$\Delta = \frac{\sqrt{4c - 1}}{2}. \tag{42}$$

Using the value of b in (23), we see that Δ is a positive reel number for $m > 2$. This gives two solutions

$$\delta\theta = c_1\frac{\cos(\Delta\ln t)}{\sqrt{t}} + c_2\frac{\sin(\Delta\ln t)}{\sqrt{t}}, \tag{43}$$

which shows that $\delta\theta \to 0$ and the shape modulus is stabilized.

For δY and δZ, one can repeat the same analysis and get equation (40) for the corresponding power k, where the constant c is the negative of the total number multiplying $\delta Y/t^2$ or $\delta Z/t^2$ in the right hand side of the linearized equation. It is necessary for stabilization that $\operatorname{Re} k \geq 0$ and by (40) this happens when

$$c \geq 0. \tag{44}$$

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From (34), we see that $c = 2N_s/(b^2 R_0) + 4N_m/(b^2 R_0^3) > 0$ for δZ and thus the fraction modulus R_1/R_2 is stabilized. Eq. (33) shows that the volume modulus $R_1 R_2$ is stabilized when $m = 3$ since the corresponding constant $c = 6N_m/(b^2 R_0^3) > 0$. To stabilize $R_1 R_2$ for $m > 3$ one should impose that $N_m > R_0^2(m-3)N_s/(6m+6)$. This condition arises due to an effect which was pointed out in [9], i.e. (depending on the dimension of the observed space) partially wrapped branes force the transverse compact dimensions to expand. The condition on the number densities avoids the presence of an instability. Up to this complication, which occurs when $m > 3$, all moduli fields in this toy model are stabilized by brane winding and momentum modes. Ignoring the string gas, i.e. setting $N_s = 0$, we find that all fields are still stabilized by membrane winding and momentum modes. Neglecting, on the other hand, the membrane gas by setting $N_w = 0$, δY and thus the volume modulus becomes unstable. Focusing on the shape modulus, we see that either the momentum or the winding stress is capable of dynamically stabilizing $\delta\theta$.

Therefore we see that the negative terms which appear in the right hand sides of (33)-(35) are responsible for the stabilization of the moduli fields. It seems that the membrane winding modes characterized by N_w does not play a role in the stabilization since N_w does not appear in the right hand sides. This is only true at the linearized level for δR_1 and δR_2, and the membrane winding modes help for the stabilization of the radii nonlinearly [32]. However the right hand side of (35), which is $(\hat{\mu}, \hat{\nu}) = (\hat{1}, \hat{2})$ component of (31), is equal to the off-diagonal component of the energy momentum tensor in the orthonormal basis $T_{\hat{1}\hat{2}}$. From (13), we see that $T_{\hat{1}\hat{2}} = 0$ for membrane winding modes exactly. Therefore, winding modes of totally wrapped branes do not play a role in the dynamics of the shape modulus. The situation is different for partially wrapped branes. From (33)-(35) we see that winding string modes, which are characterized by N_s, work for the stabilization of $\delta\theta$ and δZ, but they try to destabilize δY when $m > 3$. In the linearized approximation, the contributions of momentum modes $n_a = (1, 0)$, $n_a = (-1, 0)$, $n_a = (0, 1)$ and $n_a = (0, -1)$ are negative in the right hand sides of (33)-(35), thus they try to stabilize all moduli fields.

Although the higher level momentum modes are exponentially suppressed, one can determine their effect on the stabilization of the shape modulus. From (28), we find that

$$\delta T_{\hat{1}\hat{2}} = -N_m \frac{(n_1^4 + n_2^4)}{R_0^3 |\vec{n}|^3} \frac{\delta\theta}{(bt)^2} < 0. \tag{45}$$

for the sum of the modes (n_1, n_2) and $(n_1, -n_2)$. This term should be added to the right

hand side of (35) and since it is negative we see that the pairs of the modes (n_1, n_2) and $(n_1, -n_2)$ force $\delta\theta$ to be fixed.

IV.   CONCLUSIONS

In this work, we study the stabilization problem for the shape modulus in a toy model in Einstein gravity with gases of membranes and strings wrapping over an internal two-torus. Based on the earlier results in the literature (for strings [26–28] and for higher dimensional branes [32, 33]), it is not surprising to see that the size moduli are stabilized. Our new observation is that the winding modes of partially wrapped branes and the momentum modes give rise to stress which is capable of dynamically stabilizing the shape modulus. We also notice that winding modes of totally wrapped branes do not play a role in the dynamics of the shape modulus. Of course the model we consider is incomplete in many ways but it nicely illustrates a generic dynamical behavior. Although for a two-torus there is only one shape field to worry about, the mechanism can be generalized to work for higher dimensional tori with more than one shape moduli .

In this paper we carry the calculations in the framework of Einstein gravity. It would be interesting to generalize these findings to 10-dimensional dilaton gravity. In that case, the results would depend on the brane type. In [33], consistent field equations are obtained for a D-brane gas coupled to dilaton gravity and a natural extension of this work is to consider a dynamical shape in that toy model.

It is known that in string theory compactifications with fluxes, all of the complex structure moduli including the shape of the internal manifold can be stabilized. However, it turns out to be difficult to stabilize the volume modulus in this framework. On the other hand, volume stabilization can be achieved easily in brane gas cosmology. This motivates the inclusion of brane gases carrying fluxes and as shown in [37] all moduli fields can be stabilized in a simple toroidal compactification with a string gas and ambient flux. In this work we show that the shape moduli can be stabilized without introducing fluxes. However, one may need to introduce fluxes for other moduli (like the ones related to form-fields generating fluxes), therefore it would be interesting to develop further the framework of brane gases with fluxes.

We study the stabilization problem in a linearized setting. It would be important to extend the results to the non-linear regime without any apprxoximations. One difficulty in

this analysis is to determine the relative number densities of the momentum modes. For instance, in a square two-torus with $\theta = 0$, $n_a = (1, 0)$ mode is lighter than $n_a = (1, 1)$ mode. However, as θ increases, the energy of $n_a = (1, 1)$ mode decreases. When $\sin \theta = 1/2$, the energies of the modes $n_a = (1, 0)$ and $n_a = (1, 1)$ become equal. As θ increases more, $n_a = (1, 1)$ mode becomes the lighter one. (In [40] and [41], this relation between the shape of the torus and the energy of the momentum modes was used to weaken the experimental bounds on theories with large extra dimensions). In a non-linear analysis, fluctuations on θ can be large and it is difficult to determine the light species which would dominate the dynamics.

The torus is a special manifold where the eigenvalues of the Laplacian are known explicitly. It is worth trying to extend the arguments where this information is no longer available. In the realistic case, to any topologically nontrivial cycle, one anticipates the existence of a brane gas wrapping over that cycle. The results in brane gas cosmology suggest that in such a scenario all moduli fields should be stabilized by brane winding and momentum modes.

Acknowledgments

[1] E.W. Kolb, M.J. Perry and T.P. Walker, "Time variation of fundamental constants, primordial nucleosynthesis and the size of extra dimensions", Phys. Rev. D33 (1986) 869.

[2] R. Branderberger and C. Vafa, "Superstrings in the early universe", Nucl. Phys. B316, 391,(1989).

[3] S. Alexander , R.H. Brandenberger and D. Easson, "Brane gases in the early universe", Phys. Rev. D62, 103509, (2000), hep-th/0005212.

[4] T. Boehm and R. Brandenberger, "On T-duality in Brane Gas Cosmology", JCAP 0306 (2003) 008, hep-th/0208188.

[5] R. Brandenberger, D.A. Easson, D. Kimberly, "Loitering Phase in Brane Gas Cosmology", Nucl. Phys. B623 (2002) 421, hep-th/0109165.

[6] D. A. Easson, "Brane Gases on K3 and Calabi-Yau Manifolds", Int. J. Mod. Phys. A18 (2003) 4295, hep-th/0110225.

[7] S.H.S. Alexander, "Brane Gas Cosmology, M-theory and Little String Theory", JHEP 0310 (2003) 013, hep-th/0212151.

[8] R. Easther, B.R. Greene, M.G. Jackson and D. Kabat, "Brane gas cosmology in M-theory: late time behavior", Phys. Rev. D67 (2003) 123501, hep-th/0211124.

[9] A. Kaya and T. Rador, "Wrapped branes and compact extra dimensions in cosmology", Phys. Lett. B565 (2003) 19, hep-th/0301031.

[10] B.A. Bassett, M. Borunda, M. Serone and S. Tsujikawa, "Aspects of String-Gas Cosmology at Finite Temperature", Phys. Rev. D67 (2003) 123506, hep-th/0301180.

[11] A. Kaya, "On Winding Branes and Cosmological Evolution of Extra Dimensions in String Theory", Class. Quant. Grav. 20 (2003) 4533, hep-th/0302118.

[12] T. Biswas, "Cosmology with branes wrapping curved internal manifolds", JHEP 0402 (2004) 039, hep-th/0311076.

[13] J. Y. Kim, " Late time evolution of brane gas cosmology and compact internal dimensions", Phys. Rev D70 (2004) 104024, hep-th/0403096.

[14] A. Campos, "Asymptotic cosmological solutions for string/brane gases with solitonic fluxes", hep-th/0501092.

[15] M. Majumdar and A.-C. Davis, "D-brane Anti-brane Annihilation in an Expanding Universe", JHEP 0312 (2003) 012, hep-th/0304153.

[16] A. Campos, "Late-time dynamics of brane gas cosmology", Phys. Rev. D68 (2003) 104017, hep-th/0304216.

[17] R. Easther, B.R. Greene, M.G. Jackson and D. Kabat, "Brane gases in the early universe: thermodynamics and cosmology", JCAP 0401 (2004) 006, hep-th/0307233.

[18] A. Campos, " Late cosmology of brane gases with a two-form field", Phys. Lett. B586 (2004) 133, hep-th/0311144.

[19] R. Easther, B.R. Greene, M.G. Jackson and D. Kabat, "String windings in the early universe", hep-th/0409121.

[20] R. Danos, A.R. Frey, A. Mazumdar, "Interaction Rates in String Gas Cosmology", Phys. Rev. D70 (2004) 106010, hep-th/0409162.

[21] R. Brandenberger, D. A. Easson and A. Mazumdar, "Inflation and Brane Gases", Phys. Rev.

D69 (2004) 083502, hep-th/0307043.

[22] T. Biswas, R. Brandenberger, D.A. Easson and A. Mazumdar, "Coupled Inflation and Brane Gases", hep-th/0501194.

[23] T. Rador, "Intersection Democracy for Winding Branes and Stabilization of Extra Dimensions", hep-th/0501249.

[24] T. Rador, "Vibrating Winding Branes, Wrapping Democracy and Stabilization of Extra Dimensions in Dilaton Gravity", hep-th/0502039.

[25] T. Rador, "Stabilization of extra dimensions and the dimensionality of the observed space", hep-th/0504047.

[26] S. Watson and R. Brandenberger, "Stabilization of Extra Dimensions at Tree Level", JCAP 0311 (2003) 008, hep-th/0307044.

[27] S.P. Patil and R. Brandenberger, "Radion Stabilization by Stringy Effects in General Relativity and Dilaton Gravity", hep-th/0401037.

[28] S.P. Patil and R.H. Brandenberger, "The Cosmology of Massless String Modes", hep-th/0502069.

[29] S. Watson and R. Brandenberger, "Linear Perturbations in Brane Gas Cosmology", JHEP 0403 (2004) 045, hep-th/0312097.

[30] S. Watson, "UV Perturbations in Brane Gas Cosmology", Phys. Rev. D70 (2004) 023516, hep-th/0402015.

[31] T. Battefeld and S. Watson, "Effective Field Theory Approach to String Gas Cosmology", JCAP 0406 (2004) 001, hep-th/0403075.

[32] A. Kaya, "Volume Stabilization and Acceleration in Brane Gas Cosmology", JCAP 08 (2004) 004, hep-th/0405099.

[33] S. Arapoglu, A. Kaya, "D-brane Gases and Stabilization of Extra Dimensions in Dilaton Gravity", Phys. Lett. B603 (2004) 107, hep-th/0409094.

[34] A. Kaya, "Linear Cosmological Perturbations in D-brane Gases", JHEP 03 (2005) 03, hep-th/0502133.

[35] S. Watson, "Moduli Stabilization with the String Higgs Effect", Phys. Rev. D70 (2004) 066005, hep-th/0404177.

[36] A. Berndsen and J. Cline, "Dilaton Stabilization in brane gas cosmology", hep-th/0408185.

[37] R. Brandenberger, Y.-K. E. Cheung and S. Watson, "Moduli Stabilization with String Gases

and Fluxes", hep-th/0501032.

[38] S.P. Patil, "Moduli (Dilaton, Volume and Shape) Stabilization via Massless F and D String Modes", hep-th/0504145.

[39] V.F. Mukhanov, H. A. Feldman and R. H. Brandenberger, "Theory of Cosmological Perturbations", Phys. Rep. 215 (1992) 203.

[40] K.R. Dienes, "Shape versus Volume: Making Large Flat Extra Dimensions Invisible", Phys. Rev. Lett. 88 (2002) 011601, hep-ph/0108115.

[41] K.R. Dienes and A. Mafi, "Kaluza-Klein States versus Winding States: Can Both Be Above the String Scale?", Phys. Rev. Lett. 89 (2002) 171602, hep-ph/0207009.